

# **From Pseudo-dynamics to Real Dynamics**

H.B. Bürgi, S.C. Capelli, *Helv. Chim. Acta*, **86** (2003) 1625-1640,  
'Getting More out of Crystal-Structure Analyses'

# Results of a ‘crystal structure’ analysis

Atomic coordinates

x, y, z



Interatomic distances  
and angles



Static structure



Atomic displacement

Parameters (ADPs)

$U_{11}, U_{22}, U_{33}, U_{12}, U_{13}, U_{23}$

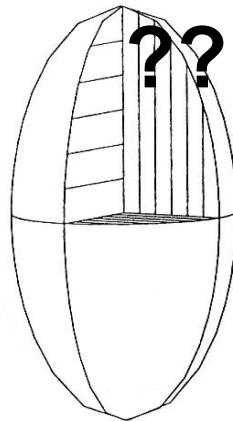


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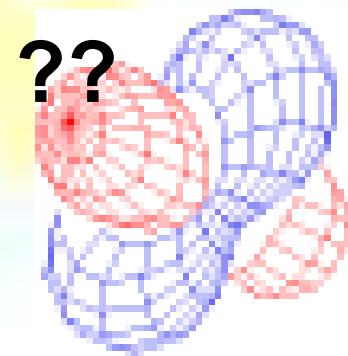


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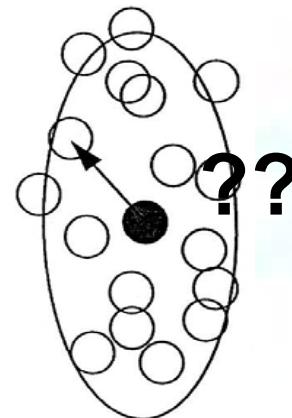
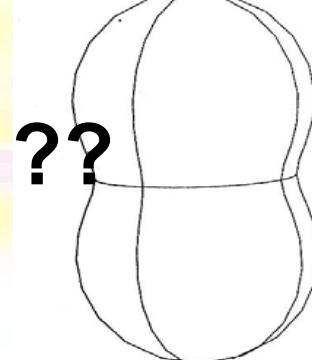
# A few reminders about ADPs



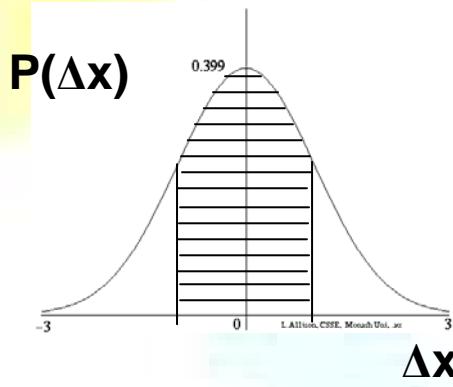
$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} & \mathbf{U}_{13} \\ \mathbf{U}_{12} & \mathbf{U}_{22} & \mathbf{U}_{23} \\ \mathbf{U}_{13} & \mathbf{U}_{23} & \mathbf{U}_{33} \end{bmatrix}$$



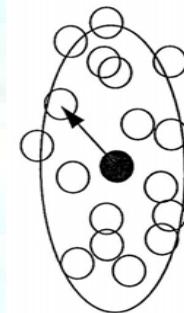
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# Instantaneous and mean-square displacements



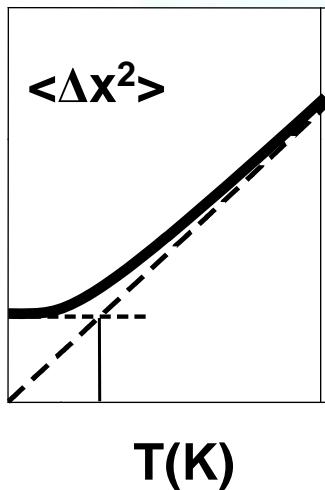
- Distribution of instantaneous atomic displacements is Gaussian



$$P(\Delta x, T) = (2\pi \langle \Delta x^2 \rangle)^{-1} \exp\{-\Delta x^2/(2\langle \Delta x^2 \rangle)\}$$

- mean square displacement amplitude is the quadratic expectation value  $\langle \Delta x^2 \rangle$

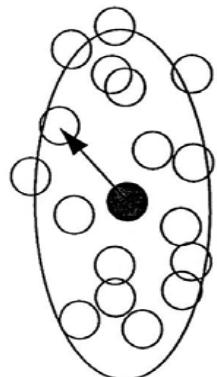
$$\langle \Delta x^2 \rangle = \int \Delta x^2 P(\Delta x, T) d\Delta x$$



- $\langle \Delta x^2 \rangle$  is temperature dependent:  
constant at very low  $T$  (zero point motion),  
proportional to  $T$  at high  $T$

$$\langle \Delta x^2 \rangle = h/(8\pi^2 m v) * \coth(h\nu/2k_B T)$$

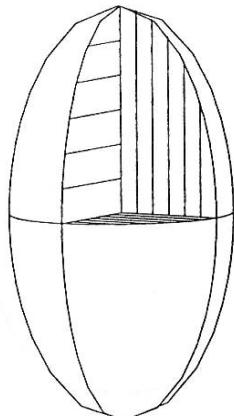
# 3-D anisotropic harmonic oscillator



$$\mathbf{v} = \Delta x \mathbf{a} + \Delta y \mathbf{b} + \Delta z \mathbf{c} = \zeta(\mathbf{a}^* \mathbf{a}) + \eta(\mathbf{b}^* \mathbf{b}) + \theta(\mathbf{c}^* \mathbf{c})$$

$$P(\mathbf{v}) = (2\pi)^{-3/2} (\det \mathbf{U}^{-1})^{1/2} \exp(-\mathbf{v}^T \mathbf{U}^{-1} \mathbf{v}/2)$$

$$\mathbf{U} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{12} & U_{22} & U_{23} \\ U_{13} & U_{23} & U_{33} \end{bmatrix} = \begin{bmatrix} \langle \zeta \zeta \rangle & \langle \zeta \eta \rangle & \langle \zeta \theta \rangle \\ \langle \zeta \eta \rangle & \langle \eta \eta \rangle & \langle \eta \theta \rangle \\ \langle \zeta \theta \rangle & \langle \eta \theta \rangle & \langle \theta \theta \rangle \end{bmatrix}$$

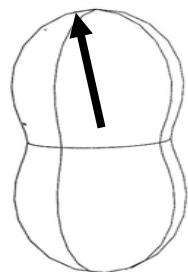


Equiprobability surface (ellipsoid)

$$-\ln P(\mathbf{v}) = \text{const} = (2\pi)^{-3/2} (\det \mathbf{U}^{-1})^{1/2} (\mathbf{v}^T \mathbf{U}^{-1} \mathbf{v})$$

# Peanuts instead of ellipsoids

$$\langle u^2(n) \rangle$$



Mean-square amplitude surface

$$\langle u^2(n) \rangle = \mathbf{n}^T \mathbf{U} \mathbf{n}$$

rms amplitude surface (PEANUT)

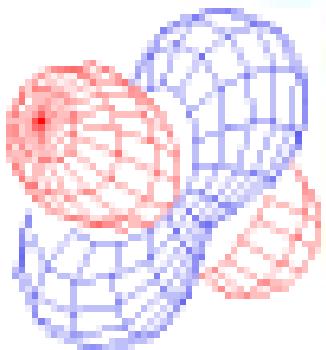
$$\langle u^2(n) \rangle^{1/2} = (\mathbf{n}^T \mathbf{U} \mathbf{n})^{1/2}$$

Mean-square amplitude difference-surface

$$\langle \Delta u^2(n) \rangle = \mathbf{n}^T (\mathbf{U}_{\text{obs}} - \mathbf{U}_{\text{model}}) \mathbf{n}$$

rms amplitude difference-surface (PEANUT)

$$\langle \Delta u^2(n) \rangle^{1/2} = (\mathbf{n}^T (\mathbf{U}_{\text{obs}} - \mathbf{U}_{\text{model}}) \mathbf{n})^{1/2}$$



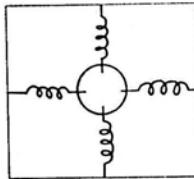
# Why bother about ADPs?

- Proper correction of interatomic distances
- Discrimination between motion and disorder
- Low frequency vibration modes (including eigenvectors)
- Specific heat curves, enthalpies and entropies
- Basis of heat conduction, thermoelectric properties, etc.
- Physically sound and practically useful model of molecular dynamics in crystals (complement to charge density model)

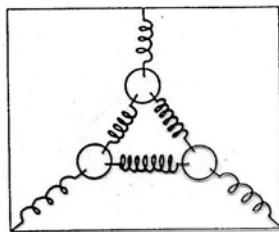
# Correlated motion

- Atomic motion in crystals is highly correlated (phonons)
- Structure analysis provides no obvious information on this correlation!
- What can be done?
- Make assumptions!?

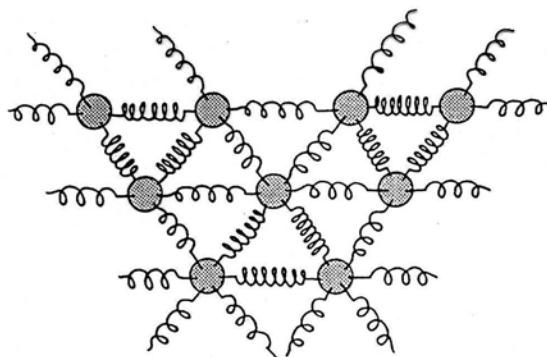
# Models of motion



Atomic Einstein or  
mean-field model



Generalized Einstein  
or molecular  
mean-field model

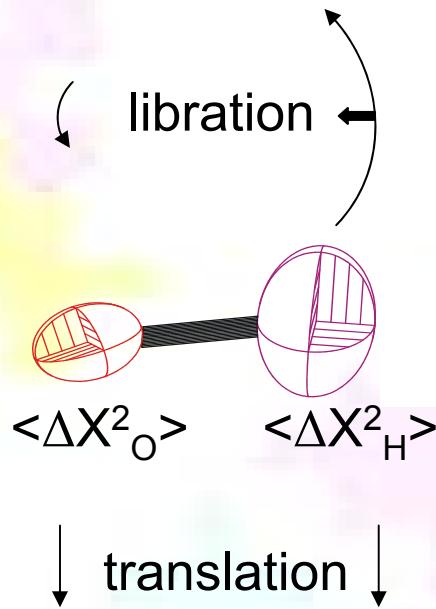


Lattice-dynamical model

# An old problem first!

... one that people have been thinking about already  
50 years ago

Interatomic distance in  
a diatomic fragment



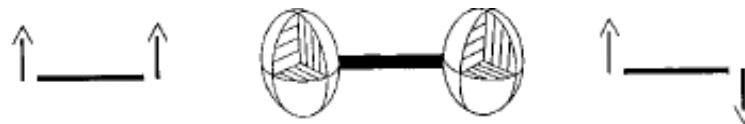
# Bond Length Corrections

$$\Delta d = \langle \Delta X^2 \rangle / (2d_{\text{obs}})$$

W.R. Busing, H.A. Levy,  
*Acta Cryst* **17** (1964) 142

$\langle \Delta X^2 \rangle$ [Å <sup>2</sup> ]	$= \langle \Delta X^2_H \rangle + \langle \Delta X^2_O \rangle - 2\langle \Delta X_O \Delta X_H \rangle$
Upper Limit	$\langle \Delta X^2_H \rangle + \langle \Delta X^2_O \rangle + 2\{\langle \Delta X^2_O \rangle \langle \Delta X^2_H \rangle\}^{1/2}$
Independent Motion	$\langle \Delta X^2_H \rangle + \langle \Delta X^2_O \rangle + 0$
H riding on O	$\langle \Delta X^2_H \rangle - \langle \Delta X^2_O \rangle$
Lower Limit	$\langle \Delta X^2_H \rangle + \langle \Delta X^2_O \rangle - 2\{\langle \Delta X^2_O \rangle \langle \Delta X^2_H \rangle\}^{1/2}$

# Diatom, coupling of atomic motions



$$\begin{bmatrix} \langle \Delta x_o^2(T) \rangle & \langle \Delta x_o \Delta x_h(T) \rangle \\ \langle \Delta x_o \Delta x_h(T) \rangle & \langle \Delta x_h^2(T) \rangle \end{bmatrix} = m^{-1/2} V \begin{bmatrix} \delta(\omega_1^{-1}, T) & 0 \\ 0 & \delta(\omega_2^{-1}, T) \end{bmatrix} V' m^{-1/2}$$

ADPs

Generalized Einstein Model

ADPs, determined  
at several T's

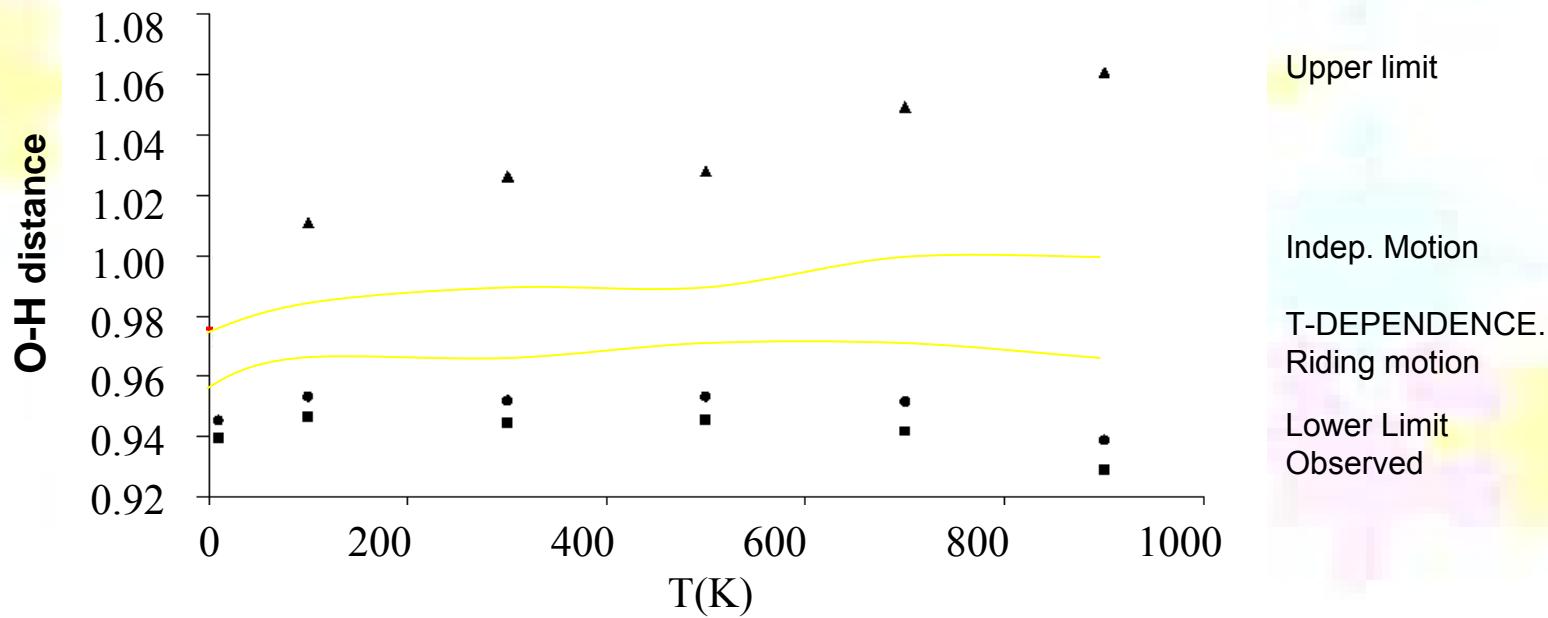
LSQ

Normal modes:  
Frequencies  $\omega$ ,  
eigenvectors  $V$   
(Mix of libration  
And translation)

Correlation ADPs  
 $\langle \Delta x_o \Delta x_h(T) \rangle$   
from model

# Comparison of Corrections

$$\langle \Delta x^2 \rangle = \langle \Delta x_O^2 \rangle + \langle \Delta x_H^2 \rangle - \text{cross term} \quad [\text{\AA}^2]$$



Average O-H distance

0.976 Å

Vibration frequencies  $\perp$  to O-H bond

888, 338 cm<sup>-1</sup>

Vibration frequencies  $\parallel$  to O-H bond

3514, 263 cm<sup>-1</sup>

# An evergreen

- Schomaker-Trueblood or  
Rigid body or TLS analysis:
- Schomaker V. & Trueblood K. N.,  
Acta Crystallogr. B24 (1968) 63-76
- > 2000 citations
- Assumption: rigid molecule without  
intramolecular motion!

# U of atom k in terms of mean square libration, translation and screw coupling motion

$$U(k) = \begin{bmatrix} U_{k,11} & U_{k,12} & U_{k,13} \\ U_{k,12} & U_{k,22} & U_{k,23} \\ U_{k,13} & U_{k,23} & U_{k,33} \end{bmatrix}$$

to be represented in terms of  $\langle t_i t_j \rangle = T_{ij}$      $\langle l_i l_j \rangle = L_{ij}$      $\langle l_i t_j \rangle = S_{ij}$

$$U_{calc}(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & r_{k3} & -r_{k2} \\ 0 & 1 & 0 & -r_{k3} & 0 & r_{k1} \\ 0 & 0 & 1 & r_{k2} & -r_{k1} & 0 \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} & T_{13} & S_{11} & S_{12} & S_{13} \\ T_{12} & T_{22} & T_{23} & S_{21} & S_{22} & S_{23} \\ T_{13} & T_{23} & T_{33} & S_{31} & S_{32} & S_{33} \\ S_{11} & S_{21} & S_{31} & L_{11} & L_{12} & L_{13} \\ S_{12} & S_{22} & S_{32} & L_{12} & L_{22} & L_{23} \\ S_{13} & S_{23} & S_{33} & L_{13} & L_{23} & L_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -r_{k3} & r_{k2} \\ r_{k3} & 0 & -r_{k1} \\ -r_{k1} & r_{k1} & 0 \end{bmatrix}$$

# Test your assumption: Rigid-bond and rigid-body tests

- calculate  $U_A$  in the direction of atom B and  $U_B$  in the direction of atom A and take the difference

$$\Delta U_{AB} = \mathbf{n}_{AB}^T (\mathbf{U}_A - \mathbf{U}_B) \mathbf{n}_{AB} \quad (|\mathbf{n}_{AB}| = 1)$$

- If A and B are connected through a covalent bond,  $\Delta U_{AB}$  is expected to be  $< 0.001 \text{ \AA}^2$ , for atoms at least as heavy as carbon (so called ‘Hirshfeld test’)
- If the  $\Delta U_{IJ}$ -values for an entire group of atoms {A, B, C, ..., Z} fulfill the Hirshfeld test, {A, B, C, ..., Z} may be considered to form a rigid body.

# Example 1: tris(bicyclo[2.1.1]hexeno)benzene

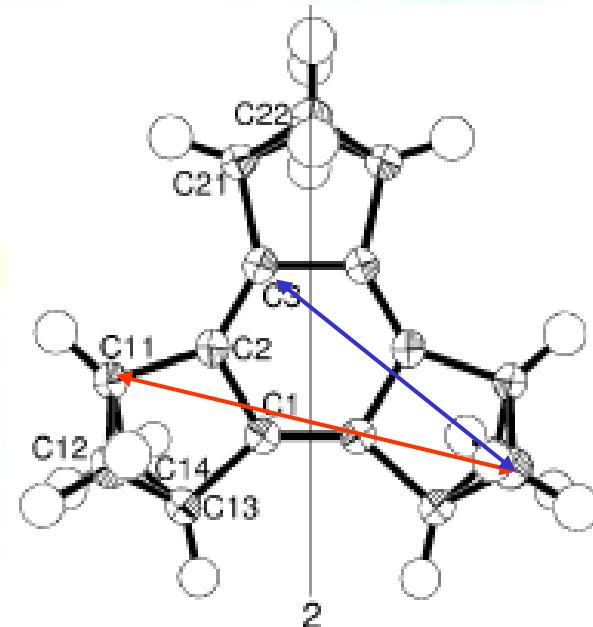
(cryst. symmetry: 2)

(mol. symmetry: 62m)

$$\langle \Delta U(\text{bonds}) \rangle = 0.0006(1) \text{ \AA}^2$$

$$\langle \Delta U(1\dots 4) \rangle = 0.0030(4) \text{ \AA}^2$$

$$\langle \Delta U(1\dots 5) \rangle = 0.0028(7) \text{ \AA}^2$$

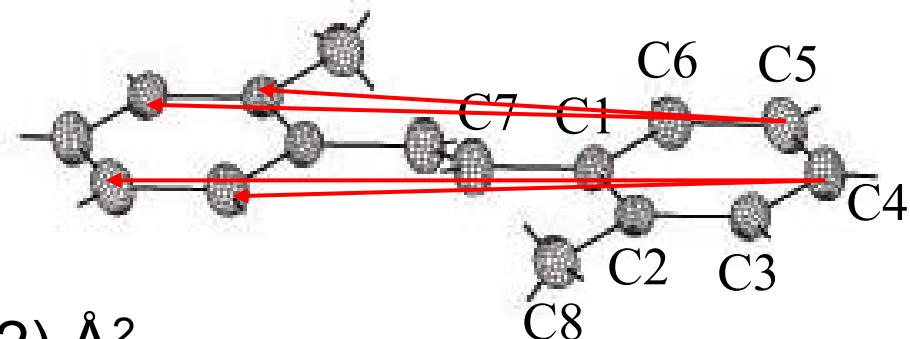


MATRIX OF DIFFS OF OBS MSDA. VALUE LISTED IS 10000\*MSDA FOR COLUMN ATOM MINUS THAT FOR ROW.

## Example 2:

### 2,2'-dimethylstilben

(symmetry: 1bar)



$$\langle \Delta U(\text{bonds}) \rangle = 0.0010(2) \text{ \AA}^2$$

$$\langle \Delta U(\text{ring...ring}) \rangle = 0.0105(2) \text{ \AA}^2$$

ATOM->	C8	C4	C7	C5	C6	C3	C2	C1	C8	C4	C7	C5	C6	C3	C2
POSN->	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1
C1	68	8	20	94	97	7	14	0	2	-1	18	8	12	5	1
C2	54	8	4	110	74	-10	0	-14	14	-3	4	-19	-17	11	
C3	68	8	14	105	78	0	10	-7	-13	-11	10	-15	-17		
C6	22-103	-76	-34	0	-78	-74	-97	18	-8	-64	4				
C5	-62-101	-72	0	34	-105-110	-94	33	-17	-28						
C7	39	-8	0	72	76	-14	-4	-20	30	-19					
C4	35	0	8	101	103	-8	-8	-8	10						
C8	0	-35	-39	62	-22	-68	-54	-68							

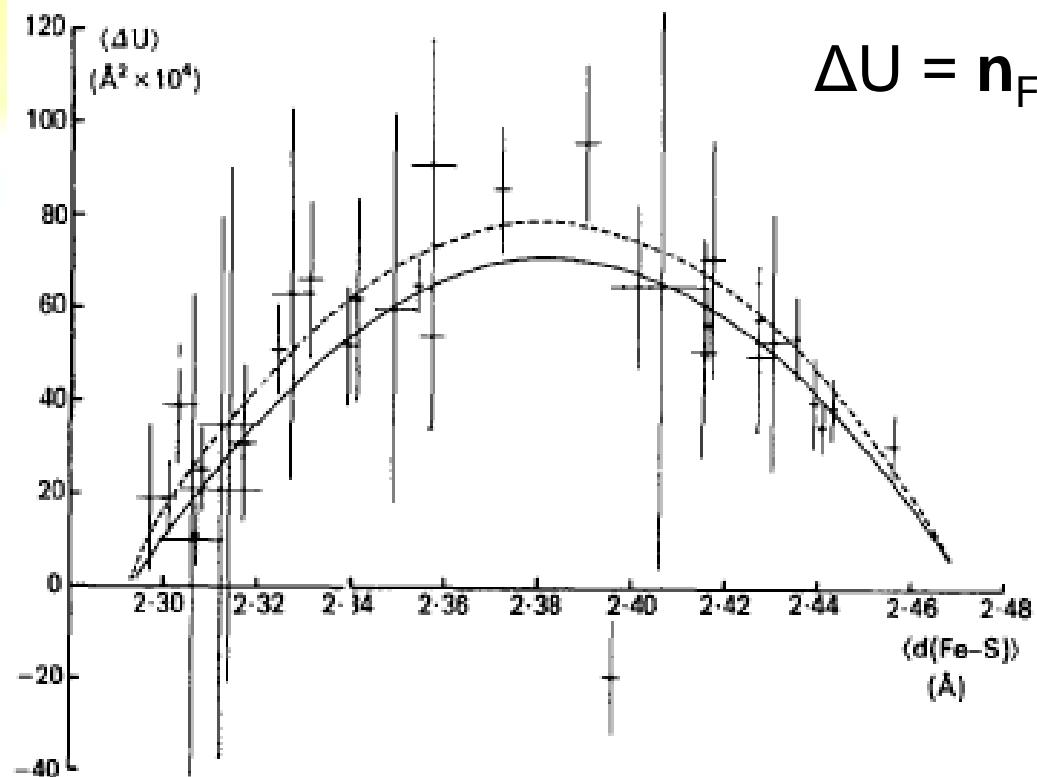
AVE. DIFF = 0.0035

R.M.S. DIFF = 0.0049

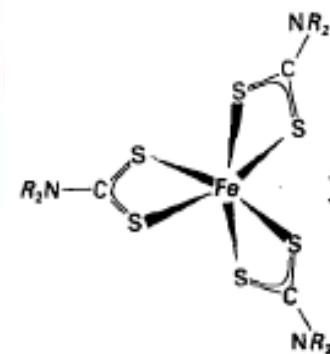
# Remarks on rigid body models

- used mostly for correction of interatomic distances.
- quality of corrections depends on the degree of rigidity and on the condition that contributions of internal molecular motion and disorder are negligibly small compared to libration and translation. Problem at low T and in disordered structures!
- the indeterminacy in  $\text{Tr}(\mathbf{S})$  does not affect these corrections, but vibrational frequencies from TLS are unreliable. No corrections of *intermolecular* distances.
- could the multi-temperature approach illustrated for diatomics be useful here?

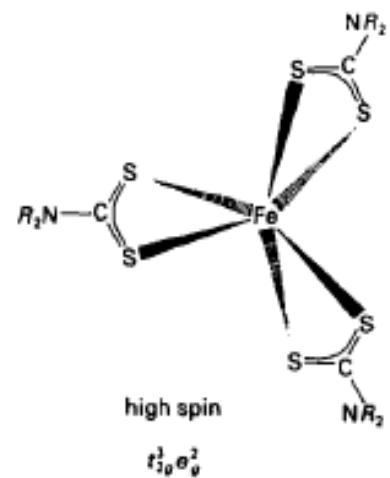
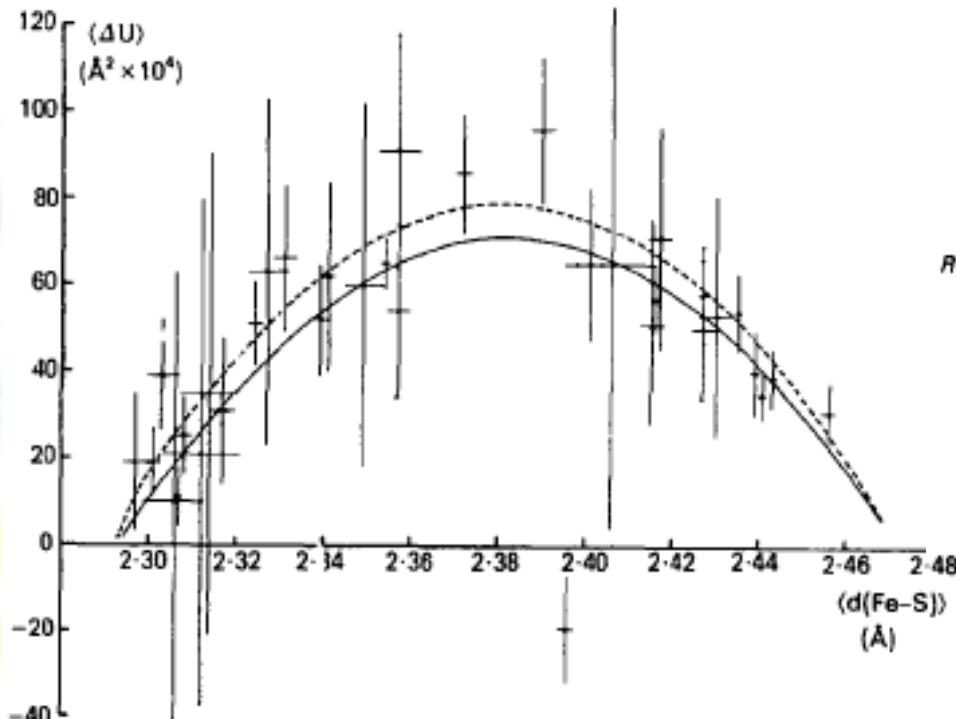
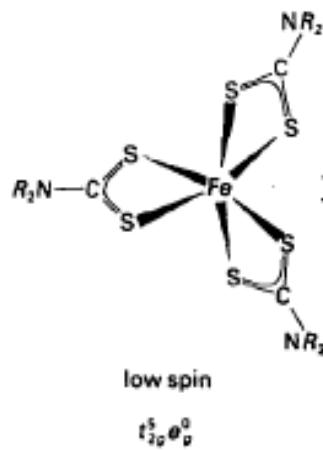
# Pathological $\Delta U$ 's from 33 iron tris-dithiocarbamates



$$\Delta U = \mathbf{n}_{\text{Fe-S}}^T (\mathbf{U}_S - \mathbf{U}_{\text{Fe}}) \mathbf{n}_{\text{Fe-S}}$$



# Spin equilibrium in iron tris-dithiocarbamates

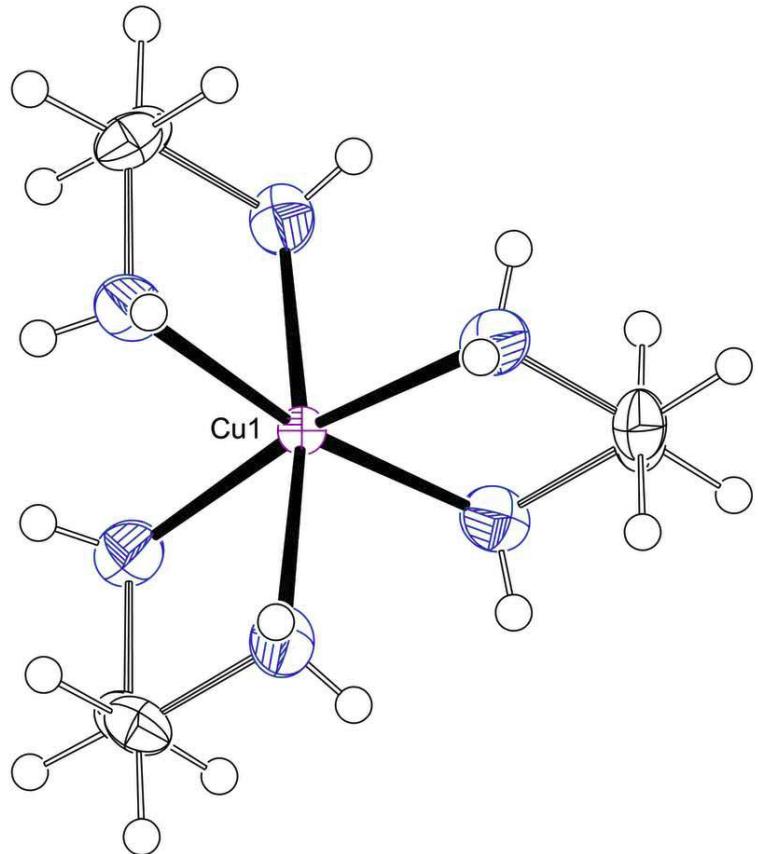


Low spin:  
 $d(\text{Fe-S})$  short

Two half S-atoms  
~0.16 Å apart

High spin:  
 $d(\text{Fe-S})$  long

# $\text{Cu(II)en}_3 \text{SO}_4$



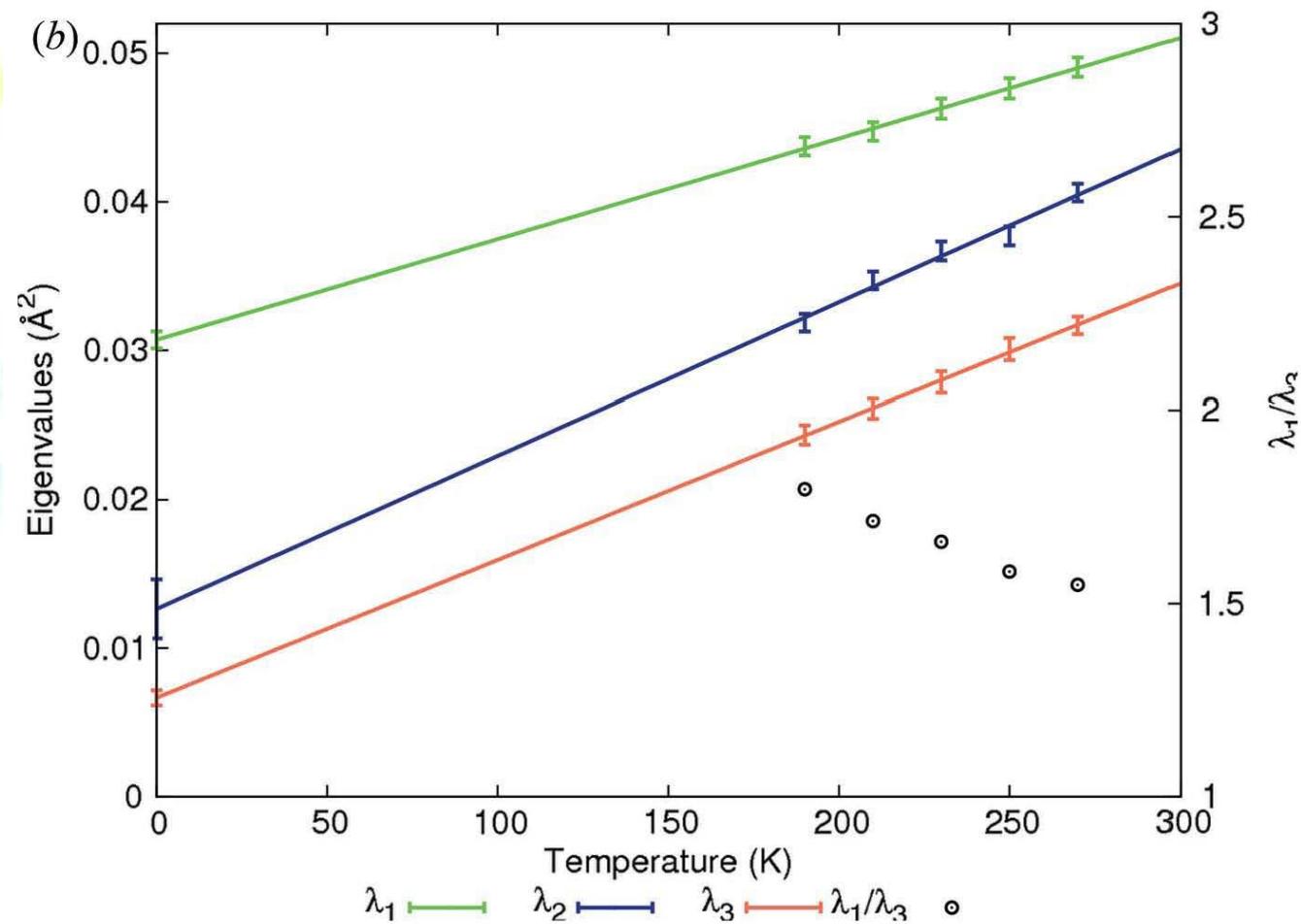
*Observed:*

Six identical, symmetry equivalent Cu(II)-N distances of length 2.15 Å

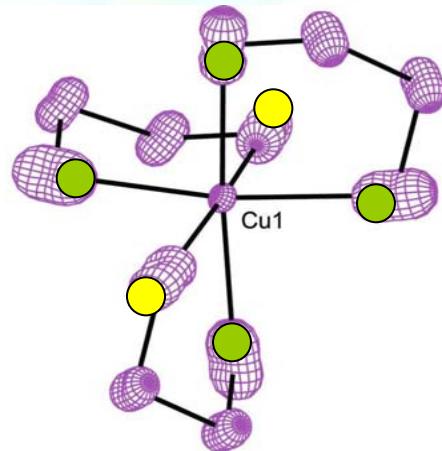
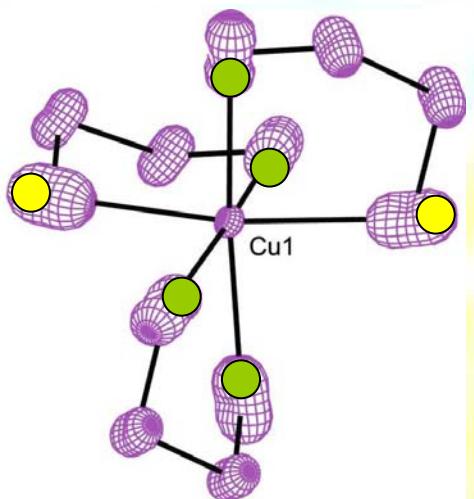
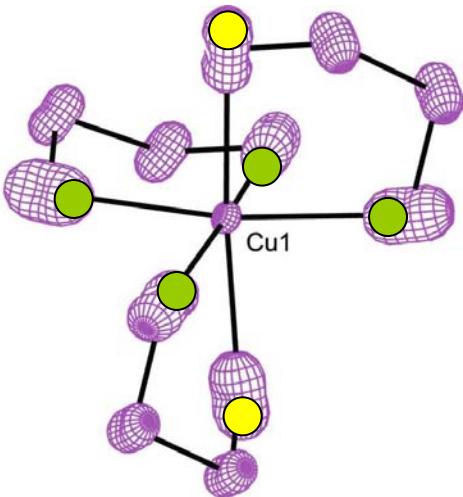
*Expected:*

Jahn-Teller distorted coordination with four short and two long Cu(II)-N distances

# Principal values of U(N) as function of T



# $\text{Cu(II)en}_3 \text{ SO}_4$



- Three ways of distorting the  $\text{Cu(II)en}_3$  ion,
- Each one occupies  $1/3$  of the unit cells in the crystal randomly
- Each one occupies a unit cell only during  $1/3$  of the observation time

# Effects absorbed by ADPs

Average over time of experiment (= time average):  
Atomic displacements arising from dynamic processes faster than hours, e.g. molecular vibrations, conformational equilibria, etc.

Average over entire crystal (= space average):  
Differences in atomic positions smaller than the resolution limit (ca. 0.5 Å) due to positional and orientational disorder

# A cautionary remark

## Crystal structure analysis

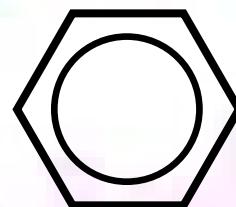
- **does not measure** the chemically interesting **bond lengths and angles**, but mean atomic positions, mean square dynamic and static displacements.
- **does not provide** 'crystal structures', but a unit cell showing the distribution of atoms averaged over the time of the experiment and the space occupied by the crystal.

# Some caveats

Cases in which any interpretation of ADPs has to be taken with a grain of salt (or better: two!)

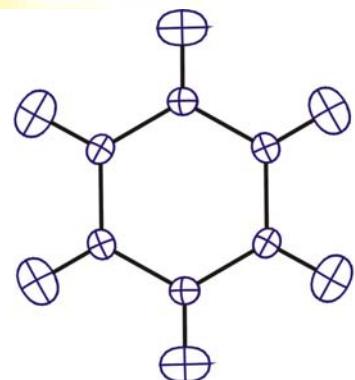
- Molecules with low-energy vibrations, e.g. torsions and angle-bends (i.e. nonrigid molecules!!!)
- Disorder with a good chemical explanation, e.g.
  - High spin/low spin mixtures in spin crossover compounds
  - Molecules with dynamic Jahn-Teller effects
  - Fluxional molecules in general
- Anharmonic motion: potentials are no longer quadratic, ADPs are Gaussian fits to non-Gaussian probability density functions.
- Absorption and (pseudo-)extinction, incomplete data

**What is the experimental evidence for  
the  $D_{6h}$  symmetry of benzene?**



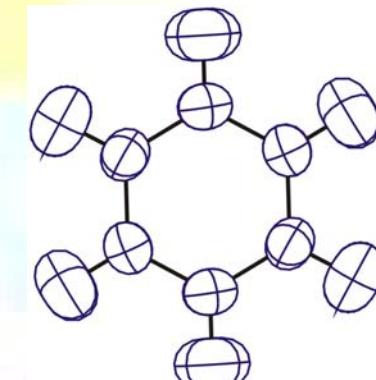
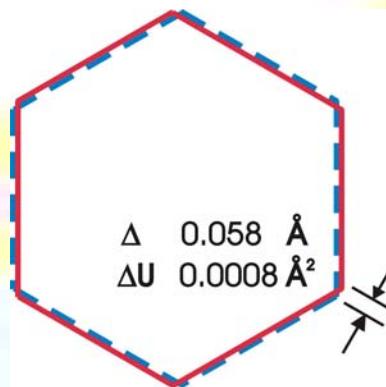
# Concerning the structure of benzene

Rms displacements  $\mathbf{U}$  of  $C_6D_6$  from neutron diffraction



15 K  
 $U_{iso}(C) \sim 0.008 \text{ \AA}^2$   
(shown: \* 2.5)

Centrosymmetric  
super-position of two  
cyclo-hexatriene  
molecules?  
(1.35 and 1.45 Å)



123 K  
 $U_{iso}(C) \sim 0.023 \text{ \AA}^2$   
(shown: \* 2.5)

G.A. Jeffrey, J.R. Ruble, R.K. Mullan, J.A. Pople, *Proc. R. Soc. London*, **A414** (1987) 47

O. Ermer, *Angew. Chem., Int. Edit.*, **26** (1987) 782

# Temperature dependence of ADPs

Vibrations of a molecule in its crystal field

$$\Sigma^x(T) = A * g * V * \delta(1/\omega, T) * V' * g' * A' + \varepsilon^x$$

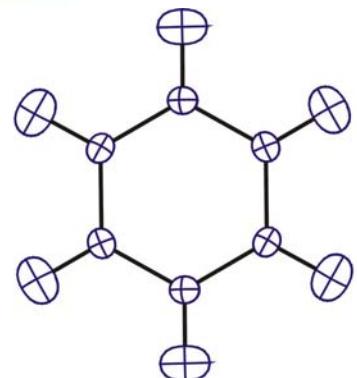
ADPs (blue) determine parameters of model (red)

ADPs,  
determined  
experimentally  
at several  
temperatures

Low frequency,  
soft vibrations ( $\omega$ ),  
e.g. librations,  
translations and  
deformations (V)

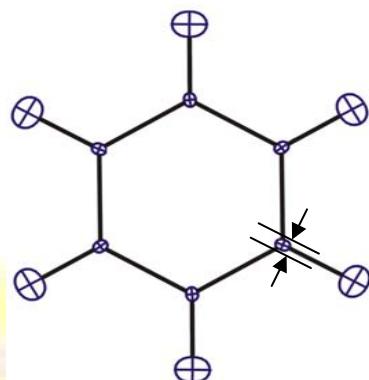
Intramolecular,  
hard vibrations  
and disorder ( $\varepsilon$ )  
(~temperature  
independent),

# Results for Benzene, C<sub>6</sub>D<sub>6</sub>



15 K

U<sub>iso</sub>(C) ~ 0.008 Å<sup>2</sup>  
(shown: \*2.5)



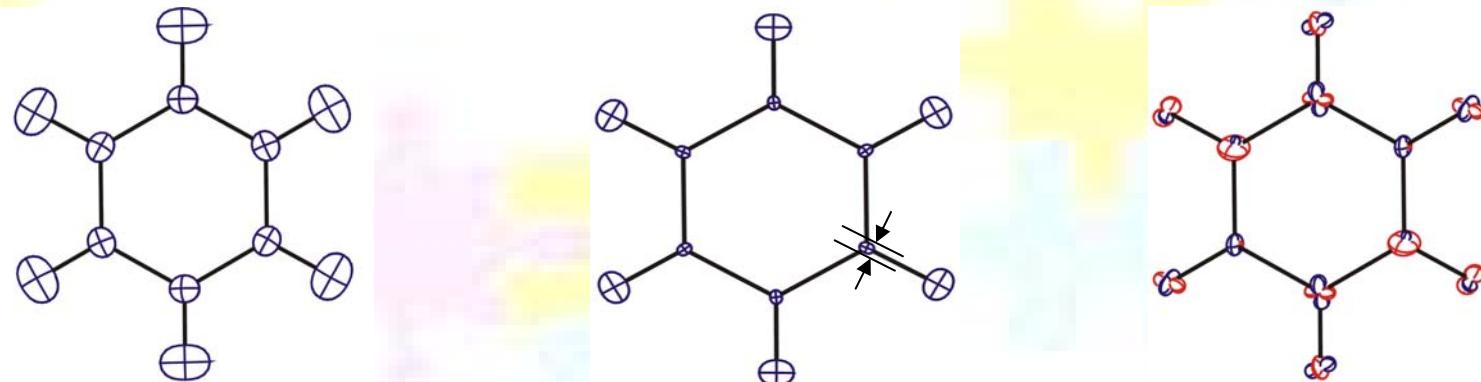
123 K

U<sub>iso</sub>(C) ~ 0.023 Å<sup>2</sup>  
(shown: \* 2.5)

$\varepsilon_C$	Along C-H	in plane	out-of-plane)
	0.0014(1)	0.0007(1)	0.0015(1) Å <sup>2</sup>

In-plane disorder contribution was estimated at 0.0008 Å<sup>2</sup>

# Results for Benzene, C<sub>6</sub>D<sub>6</sub>



Zero point motion from neutron diffraction and  
From a benchmark force field ( $\times 10^4 \text{ \AA}^2$ )

	C(bond)	C(ip)	C(oop)	D(bond)	D(ip)	D(oop)
Diffraction	14(1)	7(1)	15(1)	52(1)	83(1)	110(2)
Force Field	13	8	16	44	89	133

# Isotope effect: from ADP(D) to ADP(H)

$$\Sigma_{D^x} = A * g_D * V_D * \delta(1/\omega_D, T1) * V_D' * g_D' * A' + \varepsilon_{D^x}$$

$$\omega_D^2 = g_D * V_D * F * V_D' * g_D'$$

$$\omega_H^2 = g_H * V_H * F * V_H' * g_H'$$

$$\Sigma_{H^x} = A * g_H * V_H * \delta(1/\omega_H, T2) * V_H' * g_H' * A' + \varepsilon_{H^x}$$

Neutron diffraction  
C<sub>6</sub>D<sub>6</sub>, 15 and 123 K

Theory of normal vibrations

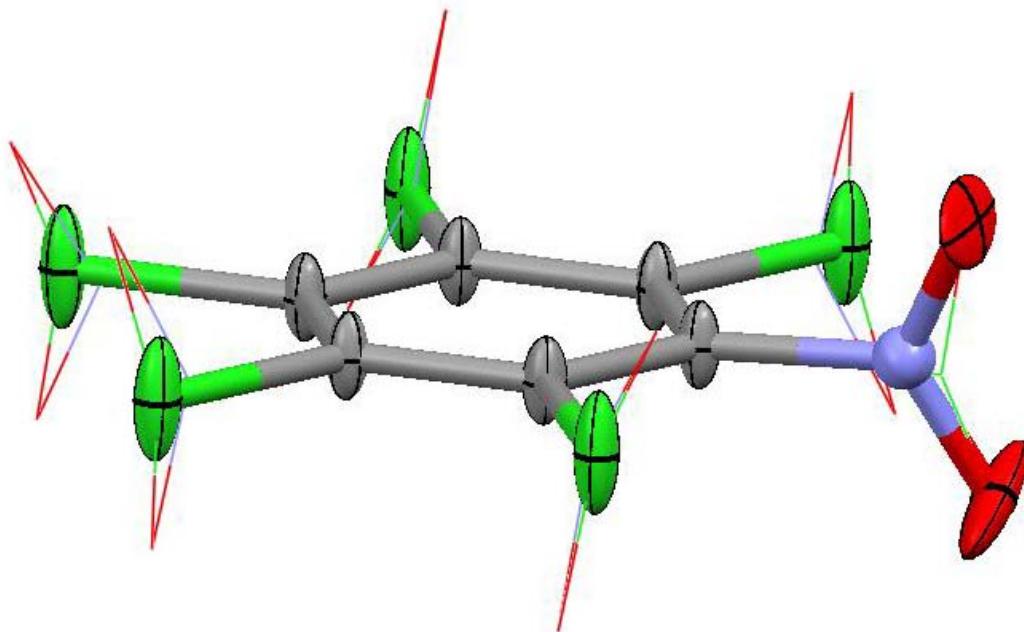
Predict ADP(C, H)  
C<sub>6</sub>H<sub>6</sub>, 110 K  
X-Ray diffraction

	$U_{11}$	$U_{22}$	$U_{33}$	$U_{12}$	$U_{13}$	$U_{23}$
C1, predicted obs-pred ( $\times 10^4 \text{ \AA}^2$ )	211 1	186 5	240 -4	13 -1	-7 -5	-9 -1
C2, predicted obs-pred ( $\times 10^4 \text{ \AA}^2$ )	195 2	236 1	222 -1	13 0	27 2	-17 -1
C3, predicted obs-pred ( $\times 10^4 \text{ \AA}^2$ )	206 5	215 0	217 -3	-17 -4	11 -1	18 2

# $\text{C}_6\text{Cl}_5\text{NO}_2$ , motion vs. disorder

Site symmetry  
3bar

Sixfold disorder



Neutron diffraction data at 5, 100, 200, 295 K <sup>a</sup>

Strong diffuse scattering <sup>b</sup>

a) J.M. Cole, H.B. Bürgi, G.J. McIntyre, Phys. Rev. B (2011) **83**, 224202

b) L. H. Thomas, T. R. Welberry, D. J. Goossens, A. P. Heerdegen, M. J. Gutmann, S. J. Teat, P. L. Lee, C. C. Wilson, J. M. Cole, Acta Cryst. (2007). **B63**, 663–673

# Static and dynamic contributions to ADPs

	$\text{C}_6\text{Cl}_5\text{NO}_2$ (5 K)	$\text{C}_6\text{D}_6$ (15 K)
$U_{\perp}(\text{C})$	0.039 Å <sup>2</sup>	0.0100 Å <sup>2</sup>
$\varepsilon(\text{C})$	0.037 (disorder)	0.0015 (oop)
dynamic	0.002	0.0085
$U_{\perp}(\text{Cl/D})$	0.078 Å <sup>2</sup>	0.028 Å <sup>2</sup>
$\varepsilon(\text{Cl/D})$	0.071 (disorder)	0.0110 (oop)
dynamic	0.007	0.0170

J.M. Cole, H.B. Bürgi, G.J. McIntyre, Phys. Rev. B (2011) **83**, 224202

L. H. Thomas, T. R. Welberry, D. J. Goossens, A. P. Heerdegen, M. J. Gutmann, S. J. Teat, P. L. Lee, C. C. Wilson, J. M. Cole, Acta Cryst. (2007). **B63**, 663–673

# Origin of dynamic contributions to ADPs

	$C_6Cl_5NO_2$ (5 K)	$C_6D_6$ (15 K)
$\omega_{\text{translation}}$	32, 35, 35 cm <sup>-1</sup>	43, 45, 51 cm <sup>-1</sup>
$\omega_{\text{libration}}$	44, 44, 44 cm <sup>-1</sup>	70, 84, 60 cm <sup>-1</sup>

Terahertz spectroscopy  $C_6Cl_5NO_2$ : band at  $\sim 40$  cm<sup>-1</sup>  
‘... attributed to molecular librations’

C. Reid, G. J. Evans, and M.W. Evans, Spectrochim. Acta A **35**, 679 (1979).

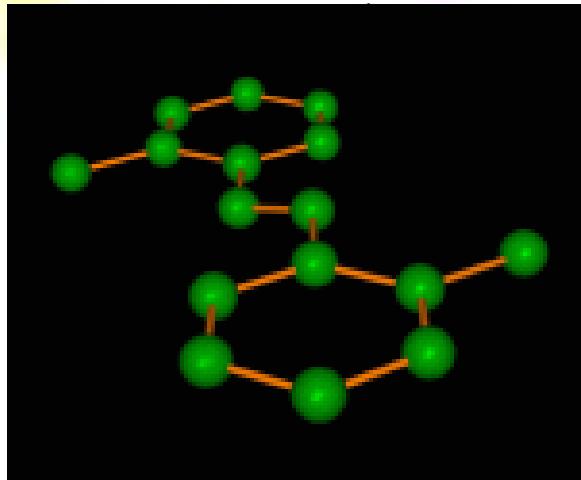
$$\frac{f_{\text{trans}}(C_6Cl_5NO_2)}{f_{\text{trans}}(C_6D_6)} = \frac{\omega^2(C_6Cl_5NO_2)}{\omega^2(C_6D_6)} \frac{M(C_6Cl_5NO_2)}{M(C_6D_6)} \approx 2$$

J.M. Cole, H.B. Bürgi, G.J. McIntyre, Phys. Rev. B (2011) **83**, 224202

# Crankshaft motion in dimethylstilbene

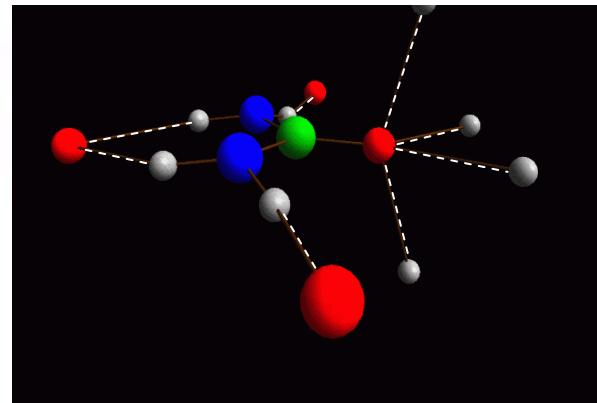
S.C. Capelli, M. Förtsch,  
H.B. Bürgi, *Acta Cryst.*  
A56 (2000) 413

Frequency  $54(2)$  cm $^{-1}$



Frequency  $45(5)$  cm $^{-1}$

T. Lüthi Nyffeler, H.B. Bürgi,  
unpublished

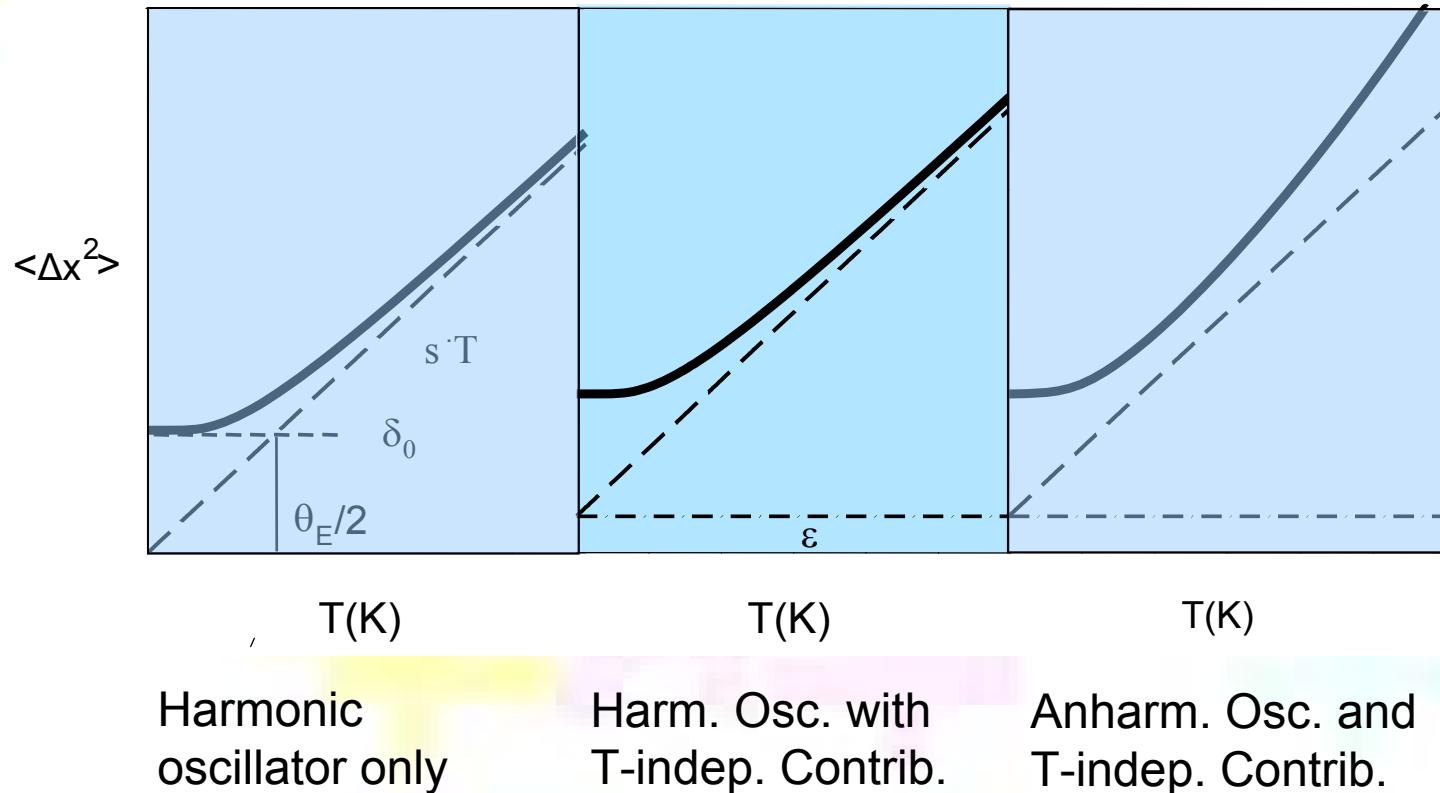


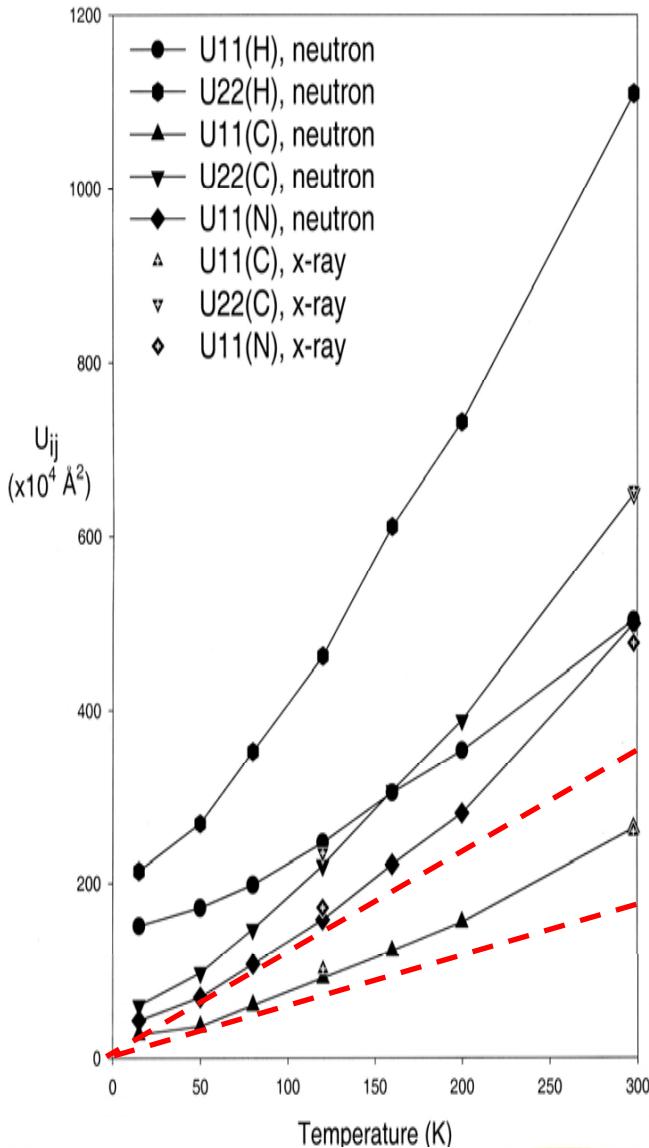
Libration and out-of-plane  
vibration of urea

# Temperature dependence of ADPs

$$\langle \Delta x^2 \rangle = \hbar / (2\omega_{\text{eff}}) \coth (\hbar \omega_{\text{eff}} / 2k_B T) + \varepsilon$$

$$\omega_{\text{eff}}(T) = \omega_0 [1 - \gamma_G \Delta V(T) / V_0]$$

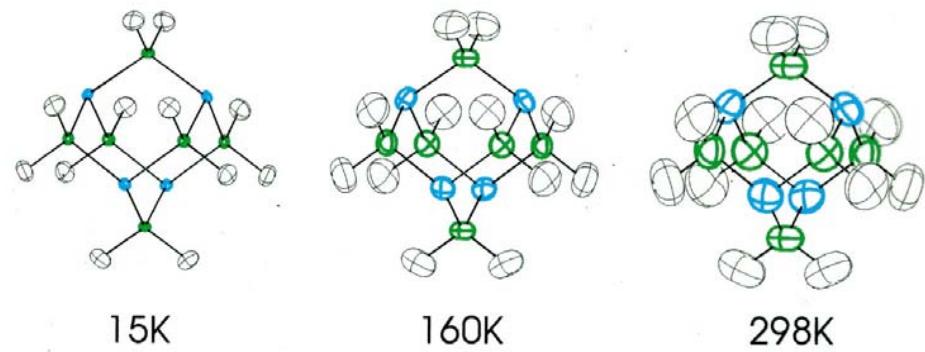




# Hexamethylenetetramine (neutron data)

Note the nonlinear  
increase of ADPs at  
higher temperatures

Anharmonicity!



# Temperature dependence of ADPs

## Quasi-harmonic model

Vibrations of a molecule in its crystal field

$$\Sigma^x(T) = A * g * V * \delta(1/\omega(T), T) * V' * g' * A' + \varepsilon^x$$

ADPs (blue) determine parameters of model (red)

ADPs,  
determined  
experimentally  
at several  
temperatures

Low frequency,  
**soft vibrations** ( $\omega$ ),  
e.g. librations,  
translations and  
deformations ( $V$ )

Intramolecular,  
**hard vibrations**  
**and disorder** ( $\varepsilon$ )  
(~temperature  
independent),

# Anharmonic motion

$$\omega_{\text{eff}}(T) = \omega_0 [1 - \gamma_G \Delta V(T) / V_0]$$

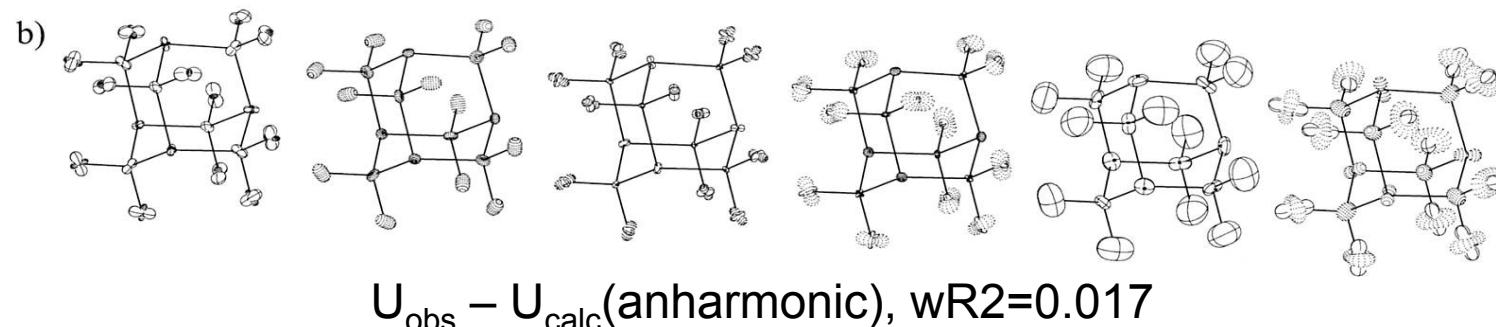
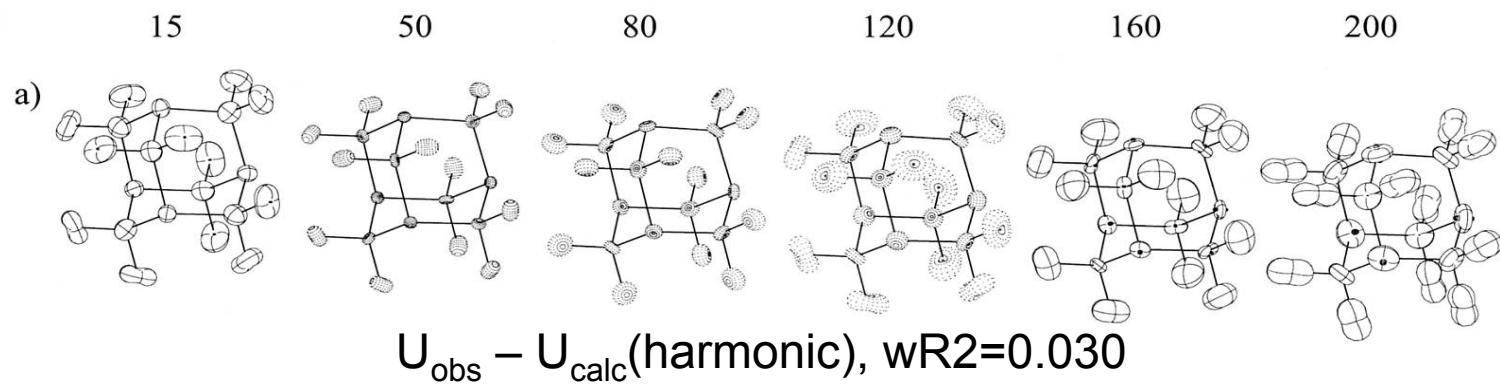
$\gamma_G$ :

$\gamma_G = 2.3$ ,

$\gamma_G = 2.2 - 2.5$ ,

Grüneisen constant  
elastic n-diffraction  
inelastic n-scattering

Quasi-harmonic model:



# $C_V$ , $C_P$ of hexamethylenetetramine

$$C_v(T) = \sum_{i=1}^3 3R \left( \frac{T}{1.437\bar{\nu}_{D,i}} \right)^3 \int_0^{1.437\bar{\nu}_{D,i}/T} \frac{x^4 e^x}{(e^x - 1)^2} dx + \\ \sum_{i=4}^6 R \left( \frac{1.437\bar{\nu}_{E,i}}{T} \right)^2 \frac{e^{1.437\bar{\nu}_{E,i}/T}}{(e^{1.437\bar{\nu}_{E,i}/T} - 1)^2} + \\ \sum_{i=7}^{3n} R \left( \frac{1.437\bar{\nu}_{int,i}}{T} \right)^2 \frac{e^{1.437\bar{\nu}_{int,i}/T}}{(e^{1.437\bar{\nu}_{int,i}/T} - 1)^2}$$

$$C_p(T) - C_v(T) = T \chi^2(T) V(T) / \kappa(T)$$

$$C_p(T) - C_v(T) = \gamma_G T \chi(T) C_v(T)$$

$$C_p(T) - C_v(T) = 3 R A_0 T C_v(T) / T_m$$

Translation

$$\nu_D = 1.5 - 1.732 \nu_E$$

Libration

Internal vibrations

B3LYP

6-311+G(2d,p)

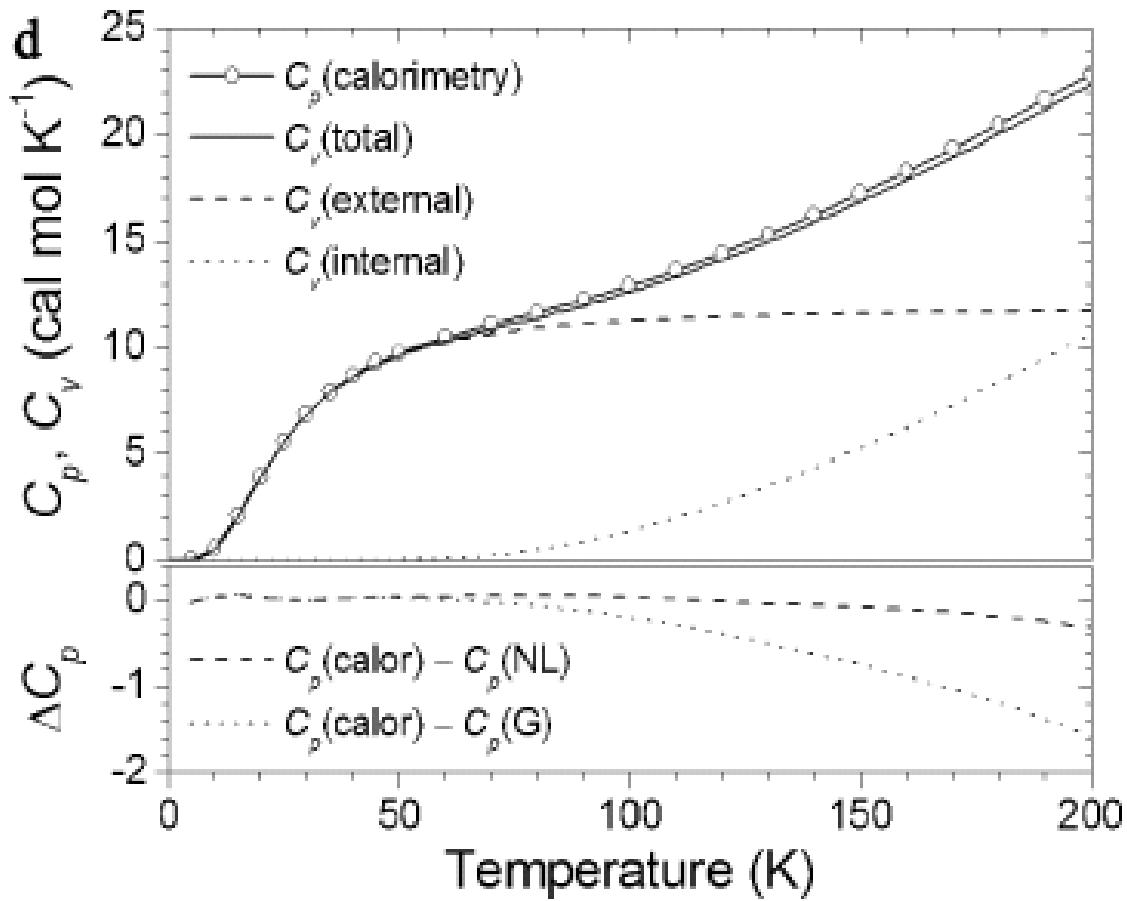
Thermodynamics

Approximations

$$A_0 = 0.0163 \text{ K mol cal}^{-1}$$

$T_m$ : melting point

# $C_V$ , $C_P$ of hexamethylenetetramine



Good agreement between calorimetric and diffraction results

Nernst-Lindemann relation is a better approximation to  $C_p - C_V$  than Grüneisen relation

Possibility to measure compressibility  $\kappa(T)$  by diffraction

# Comparing the stability of polymorphs

- $\Delta H = \int c_p dT$   $\Delta S = \int c_p/T dT$
- $\Delta G = \Delta H - T\Delta S$
- Dynamics and Thermodynamics of Crystalline Polymorphs:  
 $\alpha$ -Glycine, Analysis of Variable- Temperature Atomic  
Displacement Parameters  
J. Phys. Chem. A, **116** (2012) 8092–8099
- Dynamics and Thermodynamics of Crystalline Polymorphs. 2.  
 $\beta$ -Glycine, Analysis of Variable-Temperature Atomic  
Displacement Parameters  
J. Phys. Chem. A 117 (2013) 8001–8009
- $\gamma$ -Glycine, in preparation

Thammarat Aree et al.

# Some conclusions

- the lack of information on correlation of atomic motion can be overcome by analysing the temperature dependence of the ADPs with a normal mode approach
- more founded distance correction
- $\text{Tr}(\mathbf{S})$  can be determined
- Internal rotation can be distinguished from overall rotation
- low frequencies modes can be obtained  
(still difficult to calculate ab initio)
- in combination with information on internal vibrations crystal thermodynamic function can be calculated



**Another puzzle!**