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# ON JACQUET MODULES OF INDUCED REPRESENTATIONS OF $p$-ADIC SYMPLECTIC GROUPS

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## 1. Introduction

We fix a reductive $p$-adic group $G$. One very useful tool in the representation theory of reductive $p$-adic groups are Jacquet modules. Let us recall the definition of the Jacquet module. Let $(\pi, V)$ be a smooth representation of $G$ and let $P$ be a parabolic subgroup of $G$ with a Levi decomposition $P = MN$. The Jacquet module of $V$ with respect to $N$ is

$$V_N = V/\operatorname{span}_{\mathbb{C}}\{\pi(n)v - v; n \in N, v \in V\}.$$

Here $M$ acts in a natural way on $V_N$. We twist this action by $\delta_P^{-1/2}$, and fix such action of $M$. The Jacquet functor is exact. This functor is left adjoint to the functor of the parabolic induction, namely the Frobenius reciprocity holds

$$\operatorname{Hom}_G\left(V, \operatorname{Ind}_P^G(\sigma)\right) \cong \operatorname{Hom}_M(V_N, \sigma),$$

for a smooth representation $\sigma$ of $M$. Here $\operatorname{Ind}_P^G(\sigma)$ denotes the parabolically induced representation of $G$ by $\sigma$ from $P$ (the induction that we consider is normalized, it carries unitarizable representations to the unitarizable ones).

The Frobenius reciprocity indicates the importance of the knowledge of the Jacquet modules of representations. But the importance of the knowledge of the Jacquet modules goes far beyond the Frobenius reciprocity. The techniques of the Jacquet modules are especially convenient in the analysis of the parabolically induced representations. In this case it can be very hard or even impossible to understand a complete structure of the Jacquet module. One has a slightly weaker understanding of the Jacquet modules here: for a parabolically induced representation $\operatorname{Ind}_{P_1}^G(\sigma)$ and a parabolic subgroup $P_2 = M_2N_2$ there exists a filtration of $\left(\operatorname{Ind}_{P_1}^G(\sigma)\right)_{N_2}$

$$\{0\} = U_0 \subseteq U_1 \subseteq \ldots \subseteq U_m = \left(\operatorname{Ind}_{P_1}^G(\sigma)\right)_{N_2}$$

as an $M_2$-representation, such that one can express $U_i/U_{i-1}$, $i = 1, \ldots, m$, as certain induced representations of $M_2$ by suitable Jacquet modules of $\sigma$. The above filtration is related to the Bruhat decomposition $P_1\backslash G/P_2$. This description is done by J. Bernstein and A. V. Zelevinsky ([BZ]), W. Casselman ([C]) and Harish-Chandra.

Such a description is less than we would like to have, but still these formulas can be very useful. The reason is that one can study Jacquet modules for different parabolic subgroups, and then compare them. In this way it is possible to get pretty explicit information about induced representation. In a forthcoming paper we are going to explain these ideas in more detail.

Still, the above mentioned formulas are pretty complicated to apply in analysis of induced representations, in particular when one studies whole families of groups. So one may pose a problem of more explicit description of Jacquet modules of induced representations. For a longtime, I have been interested in a better understanding of these formulas. Let me mention that also J. Bernstein, P. Deligne and D. Kazhdan in their paper on trace Paley-Wiener theorem for reductive $p$-adic groups ([BDK]), raised the question of better understanding of the combinational structure related to the above problem (of describing Jacquet modules of induced representations).

In this paper we are going to explain one possible approach to the above problems for symplectic groups. A joint work ([ST]) with P. Sally, related more to $GSp(2)$, shows a usefulness of such an approach.

The structure describing semi-simplifications of Jacquet modules of induced representation for $GL(n)'$s was obtained by Bernstein and Zelevinsky ([Z1]). This is a Hopf algebra structure. Since we need some notation from $GL$-case, we will start with it. This case is also good motivation for the symplectic case.

In the sixth section we give one application of this approach to the construction of square integrable representations. The seventh section contains an application to the reducibility problems.

The complete proofs of results presented in this paper will appear elsewhere.

## 2. General linear groups

By $F$ we will denote a $p$-adic field. We will assume that characteristic of $F$ is different from two. Usually we will write $GL(n)$ for $GL(n, F)$.

For two smooth representations, $\pi$ of $GL(n)$ and $\tau$ of $GL(m)$, we denote

$$\pi \times \tau = \mathrm{Ind}_P^{GL(m+n)}(\pi \otimes \tau)$$

(see ([BZ])). Here $P$ denotes the parabolic subgroup

$$\left\{ \left[ \begin{array}{c|c} g & * \\ \hline 0 & h \end{array} \right] ; g \in GL(n), h \in GL(m) \right\}$$

The Grothendieck group of the category of all finite length smooth representations of $GL(m)$ is denoted by $R_m$. Recall that $R_m$ is a free $\mathbb{Z}$-module over the set of equivalence classes of all irreducible smooth representations of $GL(m)$. In a natural way we can define $\mathbb{Z}$-bilinear mapping defined on the basis

$$\begin{aligned} \times : R_m \times R_m &\to R_{m+m} \\ (\pi, \tau) &\to s.s.\,(\pi \times \tau) \end{aligned}$$

where $s.s.(\pi \times \tau)$ denotes the semi-simplification of $\pi \times \tau$. Set $R = \oplus_{n \geq 0} R_m$. Then we have graded ring structure on $R$. We can define an additive mapping $m : R \otimes R \to R$ determined by the formula

$$m(\pi \otimes \tau) = \pi \times \tau.$$

For the difference of the above operation coming from induction, the following operation is coming from Jacquet modules. Let $\sigma$ be a finite length, smooth representation of $GL(n)$ and $0 \leq p \leq n$. Then

$$r_{(p,n-p),n}(\sigma)$$

will denote the Jacquet module of $\sigma$ with respect to the parabolic subgroup.

$$\left\{ \left[ \begin{array}{c|c} g & * \\ \hline 0 & k \end{array} \right] ; g \in GL(p), k \in GL(n-p) \right\}$$

In a natural way we may consider

$$s.s.\,(r_{(p,n-p),n}(\sigma)) \in R_p \otimes R_{n-p}.$$

Set

$$m^*(\sigma) = \sum_{p=0}^{n} s.s.(r_{(p,n-p),n}(\sigma)) \in R \otimes R.$$

One extends $m^*$ $\mathbb{Z}$-linearly to $m^* : R \to R \otimes R$ .

In the proof that $(R, m, m^*)$ is a Hopf algebra, the central property is that

$$m^* : R \to R \otimes R$$

is multiplicative, where on the right hand side we put

$$(\pi_1 \otimes \pi_2) \times (\tau_1 \otimes \tau_2) = (\pi_1 \times \tau_1) \otimes (\pi_2 \times \tau_2).$$

Note that we have in this way a simple rule how to compute semisimplification of Jacquet modules of induced representations in terms of Jacquet modules of representations with which we are inducing. Let me mention two related examples at this point.

Finite fields: Put instead $F$ a finite field $\mathbf{F}$. One gets in the same way a Hopf algebra structure on $R$. Here $m^*$ computes exactly the Jacquet modules since they are semisimple. The similar observation holds for $m$.

One introduces in a natural way a scalar product on $R$ such that irreducible representation form an orthonormal basis. Now Frobenius reciprocity says that $m^*$ is adjoint operator to $m$. Also, if one introduces in a natural way a cone of positive elements in $R$, then operations $m$ and $m^*$ are positive.

A. V. Zelevinsky obtained in ([Z2]) a classification of irreducible representations of $GL(n, \mathbf{F})$ modulo cuspidal representations as a structure theory of the positive Hopf algebra $R$. He showed that

$$R = \underset{\rho\,\text{cuspidal}}{\otimes} R(\rho)$$

as positive Hopf algebras. The irreducible representations in $R(\rho)$ are parametrized by partitions. For details of this nice theory one should consult Zelevinsky's book.

Division algebras: Putting instead $F$ a central division $F$-algebra $A$, one gets again a Hopf algebra structure on $R$. In our paper ([T]) we developed some important parts of the representation theory of $GL(n, A)$ using the Hopf algebra structure. We started in ([T]) from the results of P. Deligne, D. Kazhdan and M-F. Vigneras ([DKV]).

Now we go to the symplectic case.

## 3. Symplectic groups

By $J_n$ we denote $n \times n$ matrix

$$J_n = \begin{bmatrix} 00 & \dots & 01 \\ 00 & \dots & 10 \\ \vdots & & \\ 10 & \dots & 0 \end{bmatrix}.$$

Then we denote by $Sp(n, F)$ the group of all $(2n) \times (2n)$ matrices over $F$, satisfying

$${}^tS \begin{bmatrix} 0 & J_n \\ -J_n & 0 \end{bmatrix} S = \begin{bmatrix} 0 & J_n \\ -J_n & 0 \end{bmatrix}$$

(${}^tS$ denotes the transposed matrix of $S$).

Take a smooth representation $\pi$ of $GL(n, F)$ and $\sigma$ of $Sp(m, F)$. Set

$$\pi \rtimes \sigma = \mathrm{Ind}_P^{Sp(n+m)}(\pi \otimes \sigma)$$

where $P$ is a parabolic subgroup in $Sp(n + m)$ consisting of elements

$$\begin{bmatrix} g & * & * \\ 0 & h & * \\ 0 & 0 & {}^\tau g^{-1} \end{bmatrix}$$

(${}^\tau g$ denotes the transposed matrix of $g$ with respect to the second diagonal). Here $\pi \otimes \sigma$ maps the above element to $\pi(g) \otimes \sigma(h)$.

Note that such type of multiplication of representations was introduced by Faddeev in ([F]) for the finite field case.

Let $R_n(S)$ be the Grothendieck group of the category of finite length smooth representations of $Sp(n)$. Set

$$R(S) = \underset{n\geq 0}{\oplus} R_n(S).$$

Now $\rtimes$ induces

$$\rtimes : R_n \times R_m(S) \to R_{m+m}(S),$$

and further

$$\rtimes : R \times R(S) \to R(S).$$

Also, $\rtimes$ factors to

$$\mu : R \otimes R(S) \to R(S), \mu(\pi \otimes \sigma) = s.s.(\pi \rtimes \sigma).$$

For $\sigma$, a finite length representation of $Sp(n)$, and $0 \leq k \leq n$ we denote by

$$s_{(k),(0)}(\sigma)$$

the Jacquet module for the following parabolic subgroup in $Sp(n)$

$$\left\{ \begin{bmatrix} g & * & * \\ 0 & h & * \\ 0 & 0 & {}^{\tau}g^{-1} \end{bmatrix}; g \in GL(k), h \in Sp(n-k) \right\}$$

Here the Levi factor is isomorphic to $GL(k) \times Sp(n-k)$ naturally. We consider therefore

$$s.s.(s_{(k),(0)}(\sigma)) \in R_k \otimes R_{n-k}(S)$$

and define

$$\mu^*(\sigma) = \sum_{k=0}^{n} s.s.(s_{(k),(0)}\sigma)) \in R \otimes R(S)$$

Here $\mu^*$ contains semisimplifications of all Jacquet modules for maximal parabolic subgroups.

To have Jacquet modules of induced representations, it is enough to compute

$$\mu^*(\pi \rtimes \sigma)$$

for $\pi$ representation of $GL$ and $\sigma$ of $Sp$. Now we will describe a formula for $\mu^*(\pi \rtimes \sigma)$.

Define a multiplication

$$\begin{aligned} \rtimes : \quad & (R \otimes R) \times (R \otimes R(S)) \to R \otimes R(S) \\ & ((\pi_1 \otimes \pi_2), (\pi_3 \otimes \sigma)) \quad \to (\pi_1 \times \pi_3) \otimes (\pi_2 \rtimes \sigma) \end{aligned}$$

A longer calculation gives,

**Theorem 1.** *Let $\pi$ be a finite length smooth representation of $GL$ and $\sigma$ such representation of $Sp$. Put*

$$M^*(\pi) = (m \otimes 1) \circ (\sim \otimes m^*) \circ s \circ m^*(\pi)$$

*where $s(x \otimes y) = y \otimes x$ and $\sim: R \to R$ is given on the basis $\pi \longmapsto \tilde{\pi}$ (here $\tilde{\pi}$ denotes the contragredient representation of $\pi$, $\circ$ denotes the composition.) Then*

$$\mu^*(\pi \rtimes \sigma) = M^*(\pi) \rtimes \mu^*(\sigma).$$

## 4. $GSp$

By definition, $GSp(n,F)$ is a group of all $(2n)\times(2n)$ matrices over $F$ satisfying

$$ {}^tS \begin{bmatrix} 0 & J_n \\ -J_n & 0 \end{bmatrix} S = \psi(S) \begin{bmatrix} 0 & J_n \\ -J_n & 0 \end{bmatrix}, $$

for some $\psi(S)\in F^\times$. We take formally $GSp(0)=F^\times$. Standard maximal parabolic subgroups are

$$ \left\{ \begin{bmatrix} g & * & * \\ 0 & h & * \\ 0 & 0 & \psi(h)^\tau g^{-1} \end{bmatrix} ; g\in GL(k), k\in GSp(n-k) \right\}, $$

$0\le k\le n$. Again, Levi factors are naturally isomorphic to $GL(k)\times GSp(n-k)$. Therefore one may define representation $\pi\rtimes\sigma$ of $GSp(n)$ if $\pi$ is a representation of $GL(k)$ and $\sigma$ of $GSp(n-k)$, similarly as it was done for $Sp$-groups. The Grothendieck group of the category of finite length representations we denote by $R_n(G)$. Set $R(G)=\underset{n\ge 0}{\oplus} R_n(G)$.

In the same way as before we define

$$ \rtimes : R\times R(G)\to R(G). $$

Analogously we define

$$ \mu^* : R(G)\to R\otimes R(G). $$

Now a technical variation of the formula for $Sp(n)$ gives a formula for

$$ \mu^*(\pi\rtimes\sigma) $$

for $GSp$-case. We are not going to write this formula here.

For a character $\omega$ of $F^\times$, we will denote by the same letter, the character

$$ \omega\circ\mu $$

of $GSp(n)$.

## 5. An application to square integrable representations of $GSp(n)$

Let $\rho$ be a cuspidal representation of $GL(m)$. For a non-negative integer $n$ the set

$$ [\rho,\nu^n\rho]=\{\rho,\nu\rho,\dots,\nu^n\rho\} $$

is called a segment in cuspidal representations. Here

$$\nu = |det|.$$

The representation

$$\rho \times \nu\rho \times \ldots \times \nu^n\rho$$

contains a unique essentially square-integrable subquotient. This subquotient will be denoted by $\rho([\rho, \nu^n\rho])$. This classification was obtained by J. Bernstein.

Write $\rho = \nu^\alpha \rho_o$ where $\rho_o$ is unitary and $\alpha \in \mathbb{R}$. Take a cuspidal representation $\sigma$ of $GSp(r)$. Suppose that $\rho \rtimes \sigma$ reduces. Then

$$\rho_o \cong \tilde{\rho}_o \qquad \text{and} \quad \omega_{\rho_o}\sigma \cong \sigma.$$

Here $\omega_{\rho_o}$ denotes the central character of $\rho_o$.

Suppose that $(\nu\rho_o) \rtimes \sigma$ reduces. Then this representation has a unique essentially square-integrable subquotient which will be denoted by $\delta(\{\nu\rho_o\}, \sigma)$. Now, we can define representations $\delta([\nu\rho_o, \nu^n\rho_o], \sigma)$ recursively. There exists a unique subquotient of

$$(\nu^n\rho_o) \times (\nu^{n-1}\rho_o) \times \ldots \times (\nu\rho_o) \rtimes \sigma$$

denoted by $\delta([\nu\rho_o, \nu^n\rho_o], \sigma)$, such that

$$\begin{aligned}\mu^*(\delta([\nu\rho_o, \nu^n\rho_o], \sigma)) = {} & 1 \otimes \delta([\nu\rho_o, \nu^n\rho_o], \sigma) + \nu^n\rho_o \otimes \delta([\nu\rho_o, \nu^{n-1}\rho_o], \sigma) \\ & + \delta([\nu^{n-1}\rho_o, \nu^n\rho_o]) \otimes \delta([\nu\rho_o, \nu^{n-2}\rho_o], \sigma) + \ldots \\ & \ldots + \delta([\nu^2\rho_o, \nu^n\rho_o]) \otimes \delta(\{\nu\rho_o\}, \sigma) \\ & + \delta([\nu\rho_o, \nu^n\rho_o]) \otimes \sigma.\end{aligned}$$

Moreover, $\delta([\nu\rho_o, \nu^n\rho_o], \sigma)$ is essentially square integrable representation. They are obvious generalizations of the Steinberg representation.

*Remark 1.* If $(\nu^\alpha\rho_o) \rtimes \sigma$ reduces with $\alpha \neq 0, 1, -1$ then it is also possible to define a series of square integrable representations in a similar way. This situation of $\alpha \neq 0, 1, -1$ occurs. The only example known to us of such a situation was found by F. Shahidi ([S1], [S2]). We want to thank him for communicating this result to us and for the letter where he supplied all the details of the proof. In the Shahidi case $\alpha$ is $1/2, \rho_o$ is a unitary cuspidal representation of $GL(2)$ and $\sigma$ is a character of $GSp(0)$. We are using this result in ([ST]). F. Shahidi made a general approach to such reducibility questions and he gave one description of these reducibilities ([S1]).

Suppose now that $\rho_o \cong \tilde{\rho}_o$ but

$$\omega_{\rho_o}\sigma \ncong \sigma.$$

Denote

$$\delta(\{\rho_o\}, \sigma) = \rho_o \rtimes \sigma$$

This is not an essentially square integrable representation. The representation

$$\nu\rho_o \times \rho_o \rtimes \sigma$$

contains a unique essentially square integrable subquotient which will be denoted by $\delta([\rho_o, \nu\rho_o], \sigma)$. We may define $\delta([\rho_o, \nu^n\rho_o], \sigma)$ recursively: there exists a unique subquotient $\delta([\rho_o, \nu^n\rho_o], \sigma)$ of

$$(\nu^n\rho_o) \times (\nu^{n-1}\rho_o) \times \ldots \times \rho_o \rtimes \sigma$$

such that

$$\begin{aligned}\mu^*(\delta([\rho_o, \nu^n\rho_o], \sigma)) = 1 \otimes \delta([\rho_o, \nu^n\rho_o], \sigma) &+ \nu^n\rho_o \otimes \delta([\rho_o, \nu^{n-1}\rho_o], \sigma)\\ &+ \delta([\nu^{n-1}\rho_o, \nu^n\rho_o]) \otimes \delta([\rho_o, \nu^{n-2}\rho_o], \sigma) + \ldots\\ \ldots &+ \delta([\nu^2\rho_o, \nu^n\rho_o]) \otimes \delta([\rho_o, \nu\rho_o], \sigma)\\ &+ \delta([\nu\rho_o, \nu^n\rho_o]) \otimes \delta(\{\rho_o\}, \sigma)\\ &+ \delta([\rho_o, \nu^n\rho_o]) \otimes (\sigma + \omega_{\rho_o}\sigma).\end{aligned}$$

The representations $\delta([\rho_o, \nu^n\rho_o], \sigma)$ are essentially square-integrable for $n \geq 1$. F. Rodier pointed out some of the above representations in [R1], when $\rho_o$ is a character of order 2, $\sigma$ is a character and $n = 1$.

Now we have the "mixed" case.

**Theorem 2.** *Let $\rho_1, \ldots, \rho_n, \tau_1, \ldots, \tau_m$ be mutually inequivalent unitary cuspidal representations of GL-groups (possibly $m = 0$ or $n = 0$). Let $\sigma$ be a cuspidal representation of GSp. Denote by $X$ the group generated by central characters of $\rho_1, \rho_2, \ldots, \rho_n$. Suppose that*

(i) *$\rho_i \cong \tilde{\rho}_i$, $1 \leq i \leq n$, and $\tau_j \cong \tilde{\tau}_j$, $1 \leq j \leq m$ (the first condition implies card $X \leq 2^n$).*
(ii) *Card $X = 2^n$ and $\omega\rho \ncong \rho$ for any non-trivial $\omega \in X$.*
(iii) *$\nu\tau_j \rtimes \sigma$ reduces for $1 \leq j \leq m$.*

*Let $p_i$, $1 \leq i \leq n$, $q_j$, $1 \leq j \leq m$, be positive integers. Set*

$$\Delta_i = [\rho_i, \nu^{p_i}\rho_i], \Gamma_j = [\nu\tau_j, \nu^{q_j}\tau_j], 1 \leq i \leq n, 1 \leq j \leq m$$

*Then the representation*

$$\delta(\Delta_1) \times \ldots \times \delta(\Delta_n) \times \delta(\Gamma_1) \times \ldots \times \delta(\Gamma_m) \rtimes \sigma$$

*contains a unique irreducible subquotient, denoted by*

$$\delta(\Delta_1, \ldots, \Delta_n, \Gamma_1, \ldots, \Gamma_m, \sigma),$$

*which has*

$$\delta(\Delta_1) \times \ldots \times \delta(\Delta_n) \times \delta(\Gamma_1) \times \ldots \times \delta(\Gamma_m) \otimes \sigma$$

*for a subquotient of (a suitable) Jacquet module. This subquotient is essentially square integrable.*

*Remarks 2.*

(i) If $\rho_i, \tau_j$ and $\sigma$ in Theorem 2 are all characters, then it can be shown that the list of representations from Theorem 2 in this case, is equal to essentially square integrable representations described by F. Rodier in ([R1]).

(ii) There is the problem of determining the reducibility points of

$$\nu^\alpha \rho_o \rtimes \sigma$$

where $\alpha \in \mathbb{R}$, $\rho_o$ is a unitary cuspidal representation of $GL$ and $\sigma$ a cuspidal representation of $GSp$ such that $\tilde{\rho}_o \cong \rho$ and $\omega_{\rho_o}\sigma \cong \sigma$. Besides a well-known case of $GSp(1) = GL(2)$, this problem has been solved completely by $J. - L.$ Waldspurger for one intermediate parabolic subgroup of $GSp(2)$ ([W]). For the other intermediate parabolic subgroup this is done by F. Shahidi ([S1], [S2]) and F. Shahidi's paper ([S1]) covers also the Waldspurger's case. Moreover, F. Shahidi's paper studies the general quasi-split reductive groups over $F$ and contains one description of reducibility points of induced representations from maximal parabolic subgroups by cuspidal representations (see Remark 1).

(iii) If instead of condition *(ii)* in Theorem 2 one has that $\nu^{\alpha_j}\tau_j \rtimes \sigma$ reduces, $\alpha_j > 0$, $1 \leq j \leq m$, then it is possible to construct in a similar way essentially square integrable representations. We hope that such constructed representations will exhaust all essentially square integrable representations of $GSp$-groups which are subquotients of induced representations by regular cuspidal representations.

(iv) A. Moy pointed out to me that for $GSp$-groups there may be more than one irreducible square integrable subquotient in the same unramified principal series representation. Therefore, for $GSp$-groups there will exist more square-integrable representations than those described in Theorem 2 and its modification mentioned in *(iii)*. Moreover, Moy's remark motivated us to construct an example of non-regular square integrable representation which is not supported in the minimal parabolic subgroup.

## 6. Reducibility points

At the end, let us note that the formula obtained for $\mu^* \circ \mu$ can be very useful for determining the reducibility of induced representations.

*Examples.* Let $\chi$ and $\sigma$ be characters of $F^\times$

(i) For the group $GSp(n+1)$ the representation

$$\chi \rtimes \rho([\nu 1_{F^\times}, \nu^n 1_{F^\times}], \sigma)$$

is reducible if and only if

$$\chi \in \left\{ 1_{F^\times}, \nu^{\pm(n+1)} 1_{F^\times} \right\}.$$

($1_{F^\times}$ denotes the trivial character of $F^\times$).

(ii) The representation

$$\delta([\chi, \nu\chi]) \rtimes \delta([\nu 1_{F^\times}, \nu^n 1_{F^\times}], \sigma)$$

is reducible if and only if

$$[\chi, \nu\chi] \in \{[\psi, \nu\psi], [\nu, \nu^2], [\nu^n, \nu^{n+1}], [\nu^{n+1}, \nu^{n+2}]\}$$

or

$$[\chi, \nu\chi] \in \{[\nu^{-1}\psi, \psi], [\nu^{-2}, \nu^{-1}], [\nu^{-(n+1)}, \nu^{-n}], [\nu^{-(n+2)}, \nu^{-(n+1)}]\}$$

where $\psi^2 = 1_{F^\times}$

(iii) In the following example the representation is not supported in the minimal parabolic subgroup. Let $\rho$ be a cuspidal unitary representation of $GL(m)$, $m > 1$, such that $\rho \cong \tilde{\rho}$ and $\omega_\rho \neq 1_{F^\times}$. Then

$$\chi \rtimes \delta([\rho, \nu^n \rho], \sigma)$$

is reducible if and only if

$$\chi \in \{\nu^{\pm 1}\}.$$

*Remark 3.* R. Gustafson determined the reducibility points for representations of $Sp(n)$

$$(\sigma \circ det_m) \rtimes 1$$

where $\sigma$ is an unramified character ([G]).

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